Fractional Calculus & Applied Analysis
An International Journal for Theory and Applications
VOLUME 9, NUMBER 3 (2006) ISSN 1311-0454

BOOK's Report: GEOMETRICAL METHODS FOR SOLVING OF FULLY NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS

By Peter Popivanov

Book's Information: P. Popivanov, *Geometrical Methods for Solving of Fully Nonlinear Partial Differential Equations*, Published by the Union of Bulgarian Mathematicians (UBM), UBM Series "Mathematics and Its Applications", Volume **2**

ISBN-10 954-8880-24-5, ISBN-13 978-954-8880-24-4
See details and ordering information, on the Net:
http://www.math.bas.bg/smb/books/v2.htm

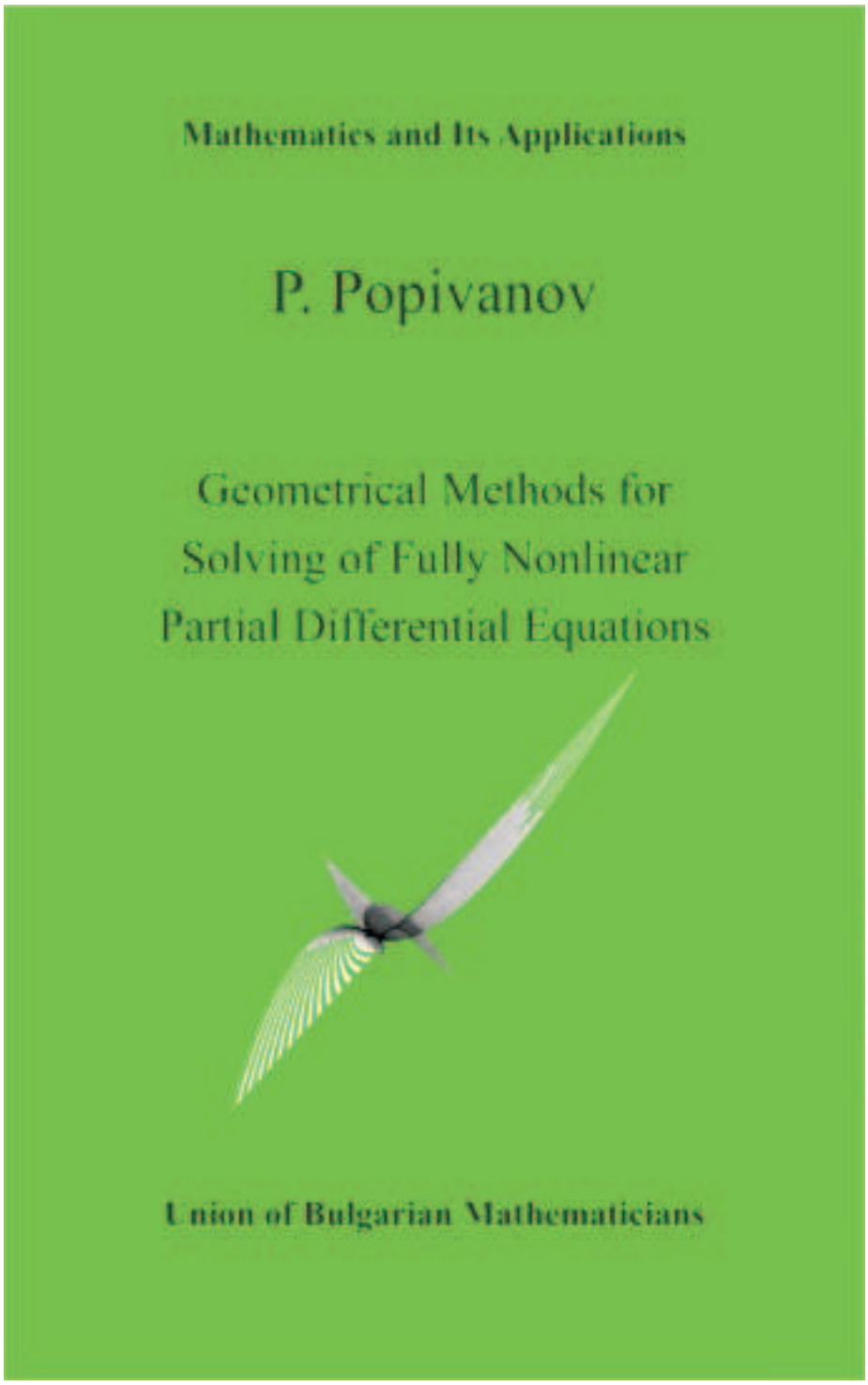
From the Preface: "This book deals with the method of characteristics for solving the Cauchy problem for first order fully nonlinear partial differential equations (PDE), for Monge-Ampere type second-order and for investigating the propagation of singularities to the solutions of several semi linear systems of PDE. It is interesting to know that anomalous singularities could appear for some non strictly hyperbolic systems. The nonlinear PDE have numerous applications in physics, mechanics, technique, biology and certainly, in the differential geometry. The link of PDE with geometry is in both directions. The solutions of corresponding equations can be considered as surfaces (integral surfaces) and we can try to develop a geometric theory of these equations, avoiding this way of a-priory estimates, fixed point theory etc. In our book we put the stress on the geometric approach, i.e. we isolate several classes of PDE (Clairaut, eikonal) whose solutions are ruled surfaces, or developable surfaces. Chapter 1 contains an elementary survey on some classical results from the differential geometry to

be used further on. A long expose on method of characteristics for solving the Cauchy problem for first-order PDE's is given. The traditional Cauchy approach and the approach via first integrals are developed. An application of the results of Chapter 2 to mechanics and geometry, including canonical transformations and Hamiltonian systems as well as the first-order PDE's satisfied with geodesics is given in Chapter 3. In Chapter 4 we prove the well known theorem of Goursat-Darboux on the existence of a local solution to the Cauchy problem. Chapters 5,6,7 contain modern results and many of them are due to the author. Chapter 5 deals with characteristics of some classes of strictly nonlinear hyperbolic systems in the plane. In Chapter 6 we study the propagation of singularities of the solutions to semi linear hyperbolic equations and systems with one space variable and we prove the appearance of new effects in comparison with linear case. The last Chapter 7 investigates the generalized Cauchy problem for non-strictly hyperbolic semi linear systems with two space variables. The Appendix contains a small survey on the catastrophe theory of R. Tom and a survey on recent results on the "generic" singularities of ruled and developable surfaces."

Audience: The first four Chapters can be used by (graduate) students in Math., Physics, Engineerings as a manual on nonlinear PDE equipped with more than 50 exercises. The rest part of the book could be of interest to PhD students and researchers in the domain of Analysis and Geometry.

Contents:

- Some auxiliary results from the differential geometry
- Cauchy method and envelope method for integration of first order fully nonlinear PDE's
- Some applications of first order nonlinear PDE's to mechanics and geometry
- Cauchy problem for Monge-Ampere type PDE's
- Characteristics of quasilinear hyperbolic systems in the hodograph plane and applications to mechanics
- Examples of anomalous singularities of the solutions to some classes of weakly hyperbolic semilnear systems in the plane
- Lorenc transformations and creation of logarithmic singularities to the solutions of some nonstrictly hyperbolic semilinear systems with two space variables
- Appendix


Mathematics and Its Applications
P. Popivanov
Geometrical Methods for Solving of Fully Nonlinear Partial Differential Equations
Union of Bulgarian Mathematicians

Report: This book offers a nice introduction to geometrical methods in fully nonlinear PDE's and their applications to model equations in mechanics and geometry. The book is nicely written and understandable, with many examples, exercises and some beautiful computer made figures and illustrations. It is very suitable as an introduction in the field and makes it useful for graduate students, scientists and mathematicians alike.

Prof. Stepan Tersian
University of Rousse, BULGARIA

About the Author: Prof. Petar Popivanov (born in 1946) is a full member of the Bulgarian Academy of Sciences and works in the Institute of Mathematics and Informatics of the Bulgarian Academy of Sciences. His publications are mainly in the domain of Partial Differential Equations (PDE) and concern linear and nonlinear microlocal analysis (hypoellipticity, local solvability, propagation of singularities), solvability of linear PDE on compact manifolds, boundary value problems for elliptic PDE (tangential oblique derivative problem, classical and viscosity solutions in the nonlinear case). He is author of about 110 papers and 2 monographs. P. Popivanov has given lectures on ODE and PDE in the Faculty of Mathematics and Informatics of the Sofia University "St. Kliment Ohridski" and in the South-West University "Neofit Rilski", and as visiting professor at many foreign universities.

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